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BRIBING AS COOPERATIVE GAME

Схема підкупу чиновників розглядається як модель кооперативної теорії ігор. На основі вектора Шеплі обчислюються так звані "істинні вартості хабара". Чисельний аналіз пояснює, чому хабарі за відкриття бізнесу такі великі, і що відбудеться, якщо хабарі стануть законними, як пропонував Г. Попов (відомий російський економіст, мер Москви за часів перебудови).
Ключові слова: коаліція, вектор Шеплі, супермодулярність, ефект снігової кулі.

Схема подкупа чиновников рассматривается как модель кооперативной теории игр. На основе вектора Шепли вычисляются так называемые "истинные значения взятки". Численный анализ объясняет, почему взятки за открытие бизнеса так высоки, и что произойдет, если взятки станут законными, как предложил Г. Попов (известный русский экономист, мэр Москвы во время перестройки).
Ключевые слова: коалиция, вектор Шепли, супермодулярность, эффект снежного кома.

The officials bribing scheme is considered as cooperative game theory model. On the base of Shapley vector so called "true bribe values" are calculated. The numerical analysis explains why the bribes for business opening are so high and what is happen, if the bribes become legitimate, as G.Popov (famous russian economist, Moscow city mayor during perestroika) proposed once.
Keywords: Coalition, Shapley vector, supermodularity, snowball effect.

Let n be number of players in cooperative game. k<n of them are called officials and the rest n-k are called businessmen. Let all businessmen make the same activity and as a result each of them creates unit surplus value. The officials couldn't create surplus value, however, the business may be made only at the condition of their mutual permission. So, the winning of coalition, which is consist of businessmen and officials equals to

W(S)=(number of businessmen in coalition)×χ(all of the officials are in coalition). (1)

Now the supermodularity property of such function will be shown. Remind, that set characteristic function is called supermodular, if for arbitrary subsets S and T such inequality is in valid

$$W(S \cup T) + W(S \cap T) \geq W(S) + W(T)\ .$$

The condition equivalent of supermodularity (however, as a rule, amenable to more easy check) is called "snowball effect". By the definition, the cooperative game has "snowball effect" if any player supply to bigger coalition ensue bigger or the same winning function increment, i.e.

$$(\forall L \subset K), (\forall i \notin K) \quad W(K \cup i) - W(K) \geq W(L \cup i) - W(L)$$

(it means, that "the bigger snowball is, the better new snow sticks").

It will be shown, than function (1) has snowball effect.

Let some player i be a businessman. Then for any coalitions couple $L \subset K$ one of three cases may take place.

Let All officials belong to L, hence they all are also belong to K, so

$$W(K \cup i) - W(K) = W(L \cup i) - W(L) = 1$$

Let not all of the officials belong to K, hence, not all of them belong to L either, so

$$W(K \cup i) - W(K) = W(L \cup i) - W(L) = 0$$

Let all of the officials belong to K, but not all of them belong to L, so

$$W(K \cup i) - W(K) = 1, \quad W(L \cup i) - W(L) = 0\ .$$

Let some player i be an official and let businessmen number at coalitions L and K (where $L \subset K$) be equal to l and k respectively, then $l \leq k$. For any coalitions couple $L \subset K$ one of three cases may take place.

Let all officials are belong to $L \cup i$, hence they all belong to $K \cup i$, so

$$W(K \cup i) - W(K) = k \geq W(L \cup i) - W(L) = l$$

2) Not all of the officials belong to $K \cup i$, hence not all of them belong to $L \cup i$ either, so

$$W(K \cup i) - W(K) = W(L \cup i) - W(L) = 0$$

3) All officials belong to $K \cup i$, but not all to $L \cup i$, so

$$W(K \cup i) - W(K) = k \geq W(L \cup i) - W(L) = 0\ .$$

So, in all of the cases snowball effect is in valid, what is equivalent to supermodularity condition. In it's turn, supermodularity is the sufficient condition, that core of the game is not empty and Shapley value belongs to core, so it's profitable to unite and to make so called grand coalition for all of the players.

Let's remind, what the Shapley value is and the way of it's calculation. For set of all players let some order to grand coalition joining be fixed (this order may be defined by permutation). Let for each player his deposit to grand coalition equals to winning function of subcoalition "with him" (as he just join the grand coalition) minus winning function of subcoalition "without him" (just before his joining the grand coalition). For example, let the grand coalition consists of four businessmen and two officials. Let some permutation be fixed, for example $(b_3, off_2, b_4, b_2, off_1, b_6, b_5)$. Note, that the first one, who makes deposit (which equals to the number of businessmen on the left of him, here 3) to grand coalition is the official positioned the most right (here off1), and each businessman after him (here b6 and b5) also makes unit deposit to grand coalition. Deposit of all players before him (both officials and businessmen) equals to zero. Of course, another players permutation generates another set of players deposits.

For each player his Shapley value is defined as his average deposit above all n! feasible players permutations. Shapley vector consists of individual Shapley values.

The total official's deposit to grand coalition is calculated as follows. Let at some permutation last (from left to right) official assume the position $k + j$, so there are j businessmen and $k - 1$ officials in the left of him. The fraction of such permutations (among all $n!$) equals to $C_{k-1+j}^{k-1} \big/ C_n^k$. At all such permutation last official takes j. By taking average among all feasible j from 0 to n-k one can get:

$$S = \sum_{j=0}^{n-k} \frac{C_{k-1+j}^{k-1}}{C_n^k} \cdot j = \frac{1}{C_n^k} \sum_{j=1}^{n-k} C_{k-1+j}^{j} \cdot j = \frac{1}{C_n^k} \sum_{j=1}^{n-k} \frac{(k-1+j)!}{(k-1)!\,j!} \cdot j = \frac{1}{C_n^k} \sum_{j=1}^{n-k} \frac{(k+(i-1))!}{(k-1)!(i-1)!} =$$

$$\frac{k}{C_n^k} \sum_{j=0}^{n-k-1} \frac{(k+i)!}{k!\,i!} = \frac{k}{C_n^k} \sum_{i=0}^{n-k-1} C_{k+i}^{i} = \frac{k}{C_n^k} C_{k+i}^{i} = \frac{k}{C_n^k} C_n^{k+1} = \frac{k}{k+1}(n-k)\ . \quad (2)$$

At sum calculation the following combinatorial identity was used. $\sum_{k=0}^{n} C_{r+k}^{k} = C_{r+n+1}^{n}$. The right hand side of (2) is the total official's deposit to grand coalition (since all officials have the same rights, then each one get the amount $\frac{n-k}{k+1}$). So, the sum, which is remaining for businessmen, equals to $(n-k) - \frac{k}{k+1}(n-k) = \frac{n-k}{k+1}$, so, each businessman get $\frac{1}{k+1}$.

Note, that each official's income equals to the total one of all of the businessmen.

It seems at first sight, that at fixed number of officials the businessmen income must increase as their number increase, cause bribes may be collected by shares among businessmen. But indeed it's not the case, and official's claim to bribe increase as number of businessmen increase at the rate, than each businessman income $\frac{1}{k+1}$ remains constant.

Conclusion. It seems that fortunately in real business bribes fraction is less, than at considered above "ideal" model. It stipulated by the fact, that corruption is still illegal and criminal punishment fear works as restriction factor. If the corruption were legitimate (or at least actually unpunishable), then it cause the situation closer to the model considered above.

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APPLYING PRINCIPLE COMPONENTS ANALYSIS FOR MODELING INVESTMENT ACCEPTANCE OF COMPANIES

Дана стаття присвячена підходу до оцінки доцільності інвестицій за допомогою факторного аналізу. Інвестори стикаються з проблемою, як систематизувати дані, обрати основні чинники та їх конфігурації, що впливають на ціну акцій компаній. В цьому випадку метод аналізу головних компонент як один з методів факторного аналізу дозволяє вирішувати таке завдання. За результатами аналізу головних компонент були складені групи українських компаній з найвищою пріоритетністю з точки зору вартості інвестицій.

Ключові слова: факторний аналіз, аналіз головних компонент, доцільність інвестицій компаній, дисперсія, факторна вага, кореляція.

Данная статья посвящена подходу к оценке целесообразности инвестиций с помощью факторного анализа. Инвесторы сталкиваются с проблемой, как систематизировать данные, выбрать основные факторы и их конфигурации, влияющие на цену акций компаний. В этом случае метод анализа главных компонент как один из методов факторного анализа позволяет решать такую задачу. По результатам анализа главных компонент были составлены группы украинских компаний с наивысшей приоритетностью с точки зрения стоимости инвестиций.

Ключевые слова: факторный анализ, анализ главных компонент, целесообразность инвестиций компаний, дисперсия, факторній вес, корреляция.

This article deals with an approach to estimation of investment acceptance by factor analysis. Investors face the problem how to systematize data, select basic factors and their configurations that influence shares price of companies. In this case Principle Components Analysis (PCA) method as one of factor analysis methods helps to solve such a task. The highest-priority groups of Ukrainian companies in terms of investment value were made according to the results of Principle Components Analysis.

Keywords: factor analysis, Principle Components Analysis, investment acceptance of companies, variance, factor's weight, correlation.

Lately investors more frequently face the task of system consideration results of Fundamental analysis, Technical analysis and Liquidity analysis in the process of making investment decision. According to every type of analysis investment object is described by the plural performances: financial ratios, performances that describe prices fluctuations on the stock market exchange, macroeconomic indicators, expert's estimations and ext. Moreover, a lot of those performances are interdependent. The professionalism of investors is expressed exactly in ability to correctly select priority factors and identify dominant configurations. In such conditions, applying factor analysis enables to determine the structure of these interconnections and provide the clench of information, explaining the plurality of indicators through a small, as a rule, number of factors. It is assumed that these factors not only provide the concentration of information but also are the most significant characteristics of investigated object. Principle Components Analysis is the most appropriate for solving such tasks.

Principal component analysis is central to the study of multivariate data. Although one of the earliest multivariate techniques it continues to be the subject of much research, ranging from new model- based approaches to algorithmic ideas from neural networks. It is extremely versatile with applications in many disciplines.

Practical application of factor analysis and directly Principle Components Analysis were researched by many prominent scientists (such as Tomashevich, 1999; Pearson, 1901; Silvester, 1889; Ayvazyan, 1989). This method was invented by Pearson (1901) and used as one of methods on diminishing data losing the least of information.

Silvester (1889) was the first who created mathematical foundation for PCA in his paper "On the reduction of a bilinear quantic of the *n-th* order to the form of a sum of *n* products by a double orthogonal substitution". Than in twelve years later Pearson (1901) proposed PCA. In many cases the "independent" variables is subject to just as much deviation or error as the "dependent" variable. Pearson (1901) observed x and y and sought the unique functional relation between them. In case he was about to deal with he supposed that the observed variables – all subject to error – to be plotted in plane, three-dimensioned or high space, and he endeavored to take a line (or plane) which will be the "best fit" to such a system of points. The method that was investigated by K. Pearson can be easily applied to numerical problems.